

PRELIMINARY OPTIMAL ORBIT DESIGN FOR THE LASER INTERFEROMETER SPACE ANTENNA (LISA)

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In this paper we present a preliminary optimal orbit analysis for the Laser Interferometer Space Antenna (LISA). LISA is a NASA/ESA mission to study gravitational waves and test predictions of general relativity. The nominal formation consists of three spacecraft in heliocentric orbits at 1 AU and trailing the Earth by twenty degrees. This configuration was chosen as a trade off to reduce the noise sources that will affect the instrument and to reduce the fuel to achieve the final orbit. We present equations for the nominal orbit design¹ and discuss several different measures of performance for the LISA formation. All of the measures directly relate the formation dynamics to science performance. Also, constraints on the formation dynamics due to spacecraft and instrument limitations are discussed. Using the nominal solution as an initial guess, the formation is optimized using Sequential Quadratic Programming to maximize the performance while satisfying a set of nonlinear constraints. Results are presented for each of the performance measures.

INTRODUCTION AND MISSION OVERVIEW

LISA is a NASA/ESA mission to detect and study gravitational waves from massive black hole systems and galactic binaries. Gravitational waves are ripples in space-time caused by massive objects. They are a prediction of general relativity but are yet to be directly detected. The detection and understanding of gravitational waves can provide breakthroughs and refinements in current relativistic theory.

LISA's primary objective is to detect gravitational waves in the frequency range of 10^{-4} to 1 Hz. This frequency range is chosen to complement ground observations for frequencies more than 100 Hz. The primary reason for using a space-based system is to eliminate gravitational noise sources. Ground based observatories are limited to high frequency measurements because of the extreme sensitivity to gravity gradient and seismic noise.

In this paper we begin by discussing the LISA mission primarily from a mission design perspective. Secondly, the science objectives and their implications on the orbit design are discussed. We consider several possible measures of performance. Next, we

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present the nominal orbit and discuss its performance under two-body and real-world orbit dynamics. Using the nominal orbit as an initial guess, a Sequential Quadratic Programming (SQP) / Projected Gradient approach is used to optimize the formation geometry while simultaneously satisfying design constraints. Results are presented for each of the performance measures considered. Finally, we discuss some conclusions and point out where more work is necessary to expand the set of acceptable solutions.

MISSION OVERVIEW

In this section we give an overview of the LISA mission. We only present a brief summary of topics that are interesting from an orbit perspective. Considerable concept development work has been performed in most areas of the mission, and for more detailed information on areas not directly related to the orbit design, we refer the reader to the LISA Final Technical Report.²

The nominal LISA formation consists of three spacecraft in carefully designed orbits. The relative positions of the spacecraft maintain a nearly equilateral triangle configuration as seen in Figure 1. The minimum science can be performed with two of the legs yet the third leg can provide additional information as well as offer redundancy in the case that one leg is lost. The primary measurement is the relative change in length of two of the legs of formation. This measurement can give information about both the direction and polarization amplitude of the gravitational wave.

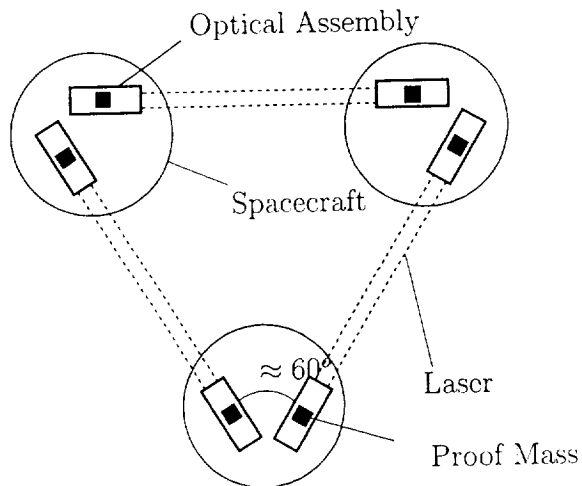


Figure 1: Equilateral Formation Configuration

Each of the three spacecraft has two optical assemblies which can be articulated so that each assembly points at the appropriate spacecraft as the formation evolves. The variation in the angle of articulation is on the order of a few degrees. Each optical

assembly can transmit and receive a laser signal. One significant difference between LISA and a conventional interferometer is that the transmitted signal is not simply reflected back. Because the leg lengths are 5 million km there is significant loss of signal strength over the traversal. To ensure adequate signal strength, the laser on the receiving end is phase locked to the incoming signal, and the phase locked signal is then transmitted back.

The leg length measurement is referenced to a free-floating proof mass internal to the spacecraft. The spacecraft's Disturbance Reduction System (DRS) is designed to ensure that no non-gravitational disturbances affect the proof mass trajectory in the measurement direction. The DRS is also responsible for ensuring that the spacecraft point in the appropriate direction. Self gravity between the spacecraft and the proof mass can cause undesirable complications in the DRS design and unacceptable measurement noise and is a topic of current research.

The target launch date for LISA is 2010 - 2012. Each spacecraft will follow an independent, optimal low-thrust trajectory to achieve the mission orbit. Once the spacecraft arrive in the mission orbits, they separate from the propulsion modules to reduce any disturbance that the module might introduce into the measurement. The nominal mission life is two years with a possibility for an extended mission of five to ten years.

ORBIT REQUIREMENTS AND PERFORMANCE

In this section we discuss several different sets of orbit performance metrics that are of interest for LISA. The orbit requirements are driven by the signal processing approach. We discuss five performance measures here and present results for each in a later section. Part of the challenge of the LISA orbit optimization is finding an optimal solution within the mission design constraints. We conclude this section with a discussion of some nonlinear constraints that each solution must satisfy for a feasible orbit.

The current approaches for LISA signal processing can be broken down into two categories. Here we present a high-level view of the approaches. For a detailed discussion see Tinto.³ The first category uses data from two of the legs in the formation. Two legs of the interferometer can provide the minimum science. In this case the third leg provides redundancy and additional measurement information. The second category involves using measurements from all three legs in the signal processing. For either category of signal processing, Doppler shifts due to relative velocities above about 15 m/s can degrade performance. So, it is necessary to ensure that the maximum rate-of-change of any leg of the formation is less than 15 m/s.

Before discussing the individual performance measures, we define a few useful variables. The position and velocity vectors of the i^{th} spacecraft are denoted \mathbf{r}_i and $\dot{\mathbf{r}}_i$ respectively. The leg formed by spacecraft i and j is denoted

$$\mathbf{L}_{ij} = \mathbf{r}_i - \mathbf{r}_j \quad (1)$$

and the leg length is simply given by

$$L_{ij} = \|\mathbf{r}_i - \mathbf{r}_j\| \quad (2)$$

The rate-of-change of the leg formed by the i^{th} and j^{th} is denoted

$$\frac{d\mathbf{L}_{ij}}{dt} = \frac{d\mathbf{r}_i}{dt} - \frac{d\mathbf{r}_j}{dt} \quad (3)$$

and the magnitude of the rate of change is simply

$$\left\| \frac{d\mathbf{L}_{ij}}{dt} \right\| = \left\| \frac{d\mathbf{r}_i}{dt} - \frac{d\mathbf{r}_j}{dt} \right\| \quad (4)$$

Two of the five performance measures considered are category one. The first measure considered, labeled cf_1 , is based on the rate-of-change of two legs of the formation. By minimizing the difference in rate-of-change between two of the legs in the formation we may be able to improve the science performance. The first cost function has the form

$$cf_1 = \frac{1}{c_1} \int_0^{t_f} \left(\left\| \frac{d\mathbf{L}_{12}}{dt} \right\| - \left\| \frac{d\mathbf{L}_{23}}{dt} \right\| \right)^2 dt \quad (5)$$

where $\dot{\mathbf{r}}_i$ is the velocity vector of the i th spacecraft, j , and k are indices for the remaining two spacecraft, and c_1 is a constant for normalization. By minimizing cf_1 , we will minimize the difference in relative velocities between two of the legs of the formation.

The second cost function considered is based on the difference in lengths of two of the legs of the formation. There are two motivations for a cost function of this form. One reason is that equal leg lengths may provide better science. The second reason is that if we achieve equal leg lengths over the entire mission, we achieve equal leg rates as well. This is because if the leg length functions are the same then so are their derivatives. This cost function is labeled cf_2 and is expressed as

$$cf_2 = \frac{1}{c_2} \int_0^{t_f} (L_{12} - L_{13})^2 dt \quad (6)$$

The remaining three cost functions are category two, and are concerned with optimizing a metric based on all three of the legs of the formation. The third cost function considered is the average rate of change of the legs of the formation. The function is labeled cf_3 and can be expressed as

$$cf_3 = \frac{1}{c_3} \int_0^{t_f} \left(\left\| \frac{d\mathbf{L}_{12}}{dt} \right\| + \left\| \frac{d\mathbf{L}_{23}}{dt} \right\| + \left\| \frac{d\mathbf{L}_{13}}{dt} \right\| \right) dt \quad (7)$$

It is also of interest to determine if we can find an orbit solution that yields all leg lengths of the formation equal in time. This solution, assuming that the design

constraints are satisfied, would minimize difficulties in the signal processing scheme as well as mitigate redundancy difficulties if one the legs of the formation is lost. The fourth cost function, cf_4 , is expressed as

$$cf_4 = \frac{1}{c_4} \int_0^{t_f} \left((L_{12} - L_{23})^2 + (L_{13} - L_{23})^2 + (L_{12} - L_{13})^2 \right) dt \quad (8)$$

where L_{12} is the length of the leg formed by spacecraft one and spacecraft two etc.

Finally, another approach to ensure that the three legs are always equal in length is to require that the interior angles of the formation are always 60 degrees. This would give a “breathing” equilateral triangle. The final cost function is labeled cf_5 and can be expressed as.

$$cf_5 = \frac{1}{c_5} \int_0^{t_f} \left((\theta_{123}(t) - \pi/3)^2 + (\theta_{231}(t) - \pi/3)^2 + (\theta_{312}(t) - \pi/3)^2 \right) dt \quad (9)$$

where θ_{123} is the angle subtended by the vectors from spacecraft 1 to spacecraft 2 and from spacecraft 2 to spacecraft 3.

The challenge of the orbit optimization is to find solutions that minimize a particular cost function and also satisfy a set of nonlinear constraints imposed by instrument and communications limitations. As mentioned previously, the largest relative velocity between any two spacecraft must be less than or equal to 15 m/s. The absolute leg lengths are also a concern. The length of the legs determines the frequency range of the instrument. The nominal leg length is chosen as 5 million km to ensure the instrument is sensitive to the desired frequencies of gravitational waves. We constrain the solutions so that the leg lengths lie within 2% of the nominal leg length. Due to articulation limitations on the spacecraft, we constrain the interior angles of the formation to lie between 58.5 degrees and 61.5 degrees. Finally, to meet communications and orbit determination requirements, the formation must be no more than twenty degrees away from the Earth, as measured from a vertex at the Sun.

Before moving on to the developement of the nominal formation, it is important to note that there are other important measures of performance that are not considered here. Other possible measures of performance include the amount of fuel required to reconfigure the formation from one optimal configuration to another in the event that one of the legs is lost. How much fuel is required to reconfigure from one optimal scenario to another is a very important question because the onboard thrusters are expected to provide a maximum of about $80\mu\text{N}$ of thrust. Hence, reconfiguring the formation could take on the order of months, or for some configurations be completely infeasible.

THE NOMINAL ORBIT

In this section we present the nominal orbit to achieve a nearly equilateral configuration using basic two-body orbit dynamics. The approach used here is based on work from Folkner¹ *et. al.*. The nominal orbit does not necessarily satisfy all of the orbit requirements. However the nominal orbit does provide a good initial guess for a numerical optimization approach. We discuss the two-body evolution of the nominal orbit as well as the evolution of the formation in the presence of real-world perturbations.

A set of orbital elements which provides a nearly equilateral formation is given below. Note that the semimajor axis a , the eccentricity, e , the inclination, i , and the argument of periapsis, ω , are the same for all orbits in the formation.

$$a = a_E \quad (10)$$

$$e = \frac{d}{2\sqrt{3}a_E} \quad (11)$$

$$i = \frac{d}{2a_E} \quad (12)$$

$$\omega = \pi/2 \quad \text{or} \quad 3\pi/2 \quad (13)$$

$$(14)$$

where a_E is the semimajor axis of the Earth's orbit about the Sun, and d is the nominal leg length. The longitude of the ascending node, Ω , and the mean anomaly, M are different for each orbit and are given by,

$$\text{for S/C 1} = (\Omega, M) \quad (15)$$

$$\text{for S/C 2} = (\Omega + 2\pi/3, M - 2\pi/3) \quad (16)$$

$$\text{for S/C 3} = (\Omega - 2\pi/3, M + 2\pi/3) \quad (17)$$

Before investigating the evolution of the nominal orbit, it is useful to define some parameters that characterize some important aspects of the motion. The average leg length over the mission life is denoted L and is given by

$$L = \frac{1}{2t_f} \int_0^{t_f} (L_{12} + L_{23}) dt \quad (18)$$

for a category one cost function and is given by

$$L = \frac{1}{3t_f} \int_0^{t_f} (L_{12} + L_{23} + L_{13}) dt \quad (19)$$

for a category two cost function. The maximum difference between legs of interest, over the mission life, is defined as Δr . Note that this quantity is defined slightly differently depending on whether the cost function is category one or two. For a cost function aimed

at optimizing a parameter based on two legs (category one), only the two legs of interest are used in determining Δr and we have

$$\Delta r = \max (L_{12}(t) - L_{23}(t)) \quad (20)$$

For a category two performance metric it is more useful to define Δr as

$$\Delta r = \max ([(L_{12}(t) - L_{23}(t)) \ (L_{12}(t) - L_{13}(t)) \ (L_{23}(t) - L_{13}(t))]) \quad (21)$$

Another useful design parameter is the maximum percent difference in leg length, $\% \Delta r$ and is given by

$$\% \Delta r = 100 \Delta r / L \quad (22)$$

The maximum difference in leg rate of change is denoted $\Delta \dot{r}$ and for a category one metric is given by

$$\Delta \dot{r} = \max \left(\left\| \frac{d\mathbf{L}_{12}(t)}{dt} \right\| - \left\| \frac{d\mathbf{L}_{23}(t)}{dt} \right\| \right) \quad (23)$$

For a category two metric we have Finally, the maximum leg rate of change is denoted \dot{r} and for a category one metric we have

$$\dot{r} = \max \left(\left\| \left\| \frac{d\mathbf{L}_{12}(t)}{dt} \right\| \right\| \left\| \frac{d\mathbf{L}_{23}(t)}{dt} \right\| \right) \quad (24)$$

and for a category two metric we have

$$\dot{r} = \max \left(\left\| \left\| \frac{d\mathbf{L}_{12}(t)}{dt} \right\| \right\| \left\| \frac{d\mathbf{L}_{23}(t)}{dt} \right\| \left\| \frac{d\mathbf{L}_{13}(t)}{dt} \right\| \right) \quad (25)$$

Figure 2 shows the evolution of several relevant parameters of the nominal solution under the gravitational influence of a spherical central body with no perturbations. The upper left hand plot shows the lengths of the legs of the formation as functions of time. The leg lengths oscillate sinusoidally centered at 5 million km with an amplitude of about 100 thousand km. In the top right plot we see the rate of change of the leg lengths over one year. The leg rates oscillate about zero m/s with a period of one year and an amplitude of about 4 m/s. In the bottom left hand plot of Figure 2, we see the evolution of the interior angles of the formation. The angles oscillate about 60° with an amplitude of about 0.45 degrees.

In table 1 we see the relevant statistics for the nominal orbit. The average arm length is about 5 million km as desired. The maximum difference between any two arm lengths is 39370 km and the maximum percent difference is 0.784%. The maximum difference in leg rates is 7.948 m/s and the maximum leg rate of change is 4.010 m/s.

Before moving on to the numerical optimization, we look at the affects of perturbations on the evolution of the nominal orbit. Figure 3 shows the evolution of some

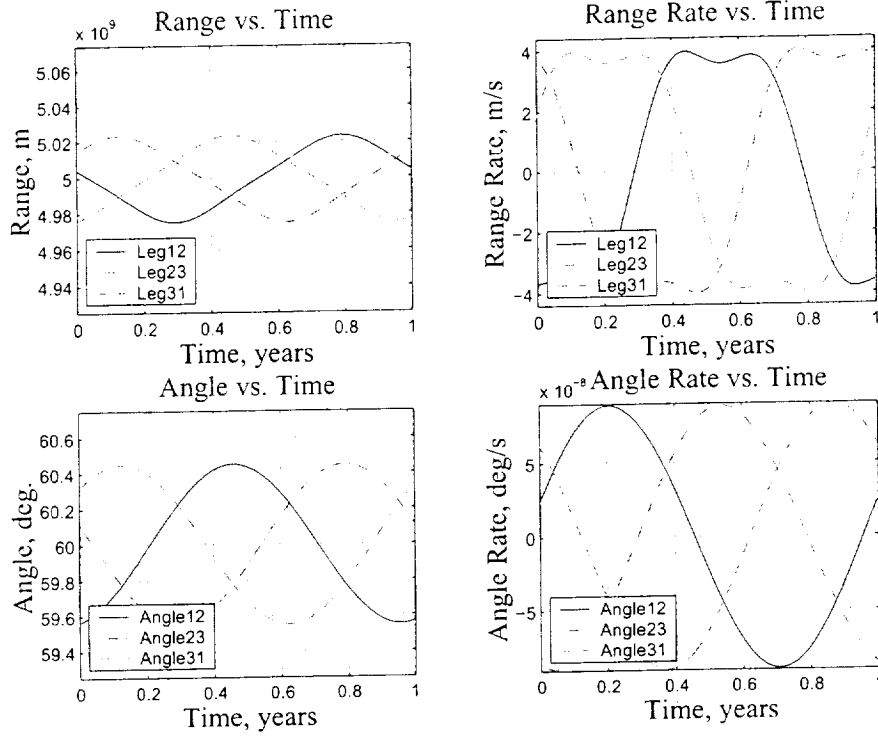


Figure 2: Nominal Unperturbed Evolution

Table 1: Statistics for the Unperturbed Orbit

Variable Name	2 Leg Definition
L	$4.999e^6$ km
Δr	39370 km
$\% \Delta r$	0.7874 %
$\Delta \dot{r}$	7.948 m/s
\dot{r}	4.010 m/s

relevant parameters when perturbations are included in the orbit force model for a five year propagation. All of the planets and the Earth's moon are included in the force model. We see that the system is unstable. In Table 2 are the relevant statistics for the nominal orbit with perturbations. The performance is poorer than the nominal two-body solution. The average arm length is still about 5 million km. However, the maximum difference between any two arm lengths is now 132,200 km and the maximum percent difference is now 2.634%. The maximum difference in leg rates is increased to 29.08 m/s and the maximum leg rate of change is 19.06 m/s. From this example, it is evident that we need modify the nominal orbit not only to improve the science performance, but also to stabilize the system for the possibility of an extended mission.

In the next section we discuss an approach to numerically optimize the performance

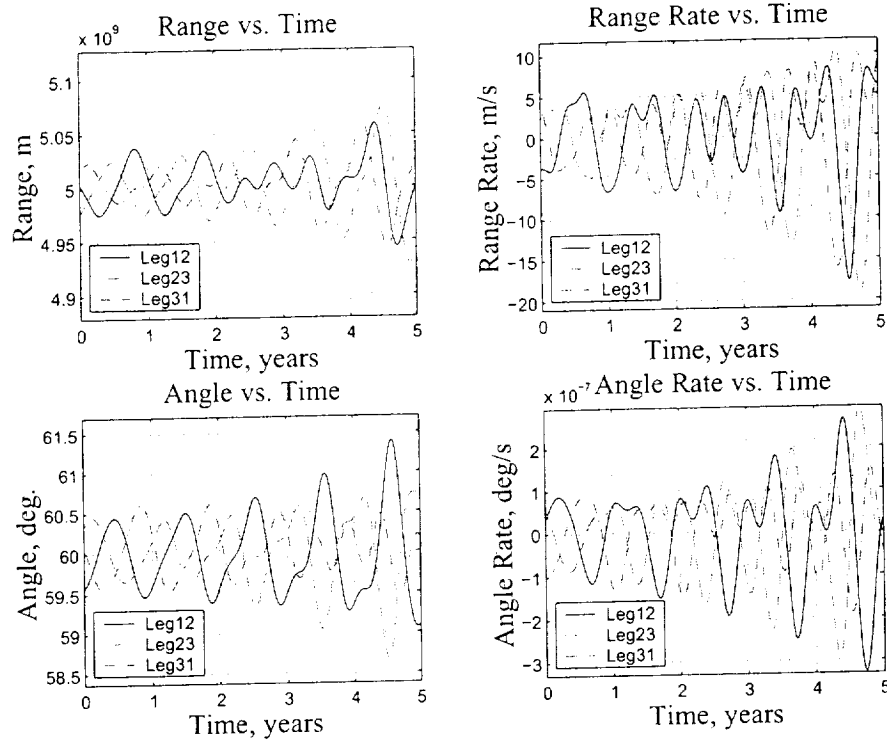


Figure 3: Nominal Evolution with Perturbations

of the formation using the nominal solution as an initial guess. The purpose of the numerical optimization is two-fold. First, we need to improve the performance of the nominal orbit, and second we need to reduce the negative affects of perturbations and if possible, use the perturbations to our advantage.

OPTIMIZATION APPROACH

Here we discuss the optimization approach and details of how the cost and constraint functions are evaluated. The optimizer used is MATLAB's "fmincon" function. The function uses a Sequential Quadratic Programming approach to solve the Kuhn-Tucker

Table 2: Statistics for the Perturbed Orbit (5 Year Propagation)

Variable Name	2 Leg Definition
L	$5.00e^6$ km
Δr	132,200 km
$\% \Delta r$	2.634 %
$\Delta \dot{r}$	29.08 m/s
\dot{r}	19.06 m/s

necessary equations. For a multivariable, non-linearly constrained problem the Kuhn-Tucker conditions are

$$\nabla f(x^*) + \sum_{i=1}^n \lambda_i^* \cdot \nabla G_i(x^*) = 0 \quad (26)$$

$$\lambda_i^* \cdot G_i(x^*) = 0 \quad i = 1, \dots, m \quad (27)$$

$$\lambda_i^* \geq 0 \quad i = m_c + 1, \dots, m \quad (28)$$

where $f(x^*)$ is the cost function evaluated at the optimum and G^* is the vector of constraint functions evaluated at the optimum. Simply stated, the necessary conditions for the constrained optimal solution are that the gradient of the cost function and the gradients of the active constraints cancel at the solution. Furthermore, the Lagrange multipliers of any inactive constraints are zero at the solution point.

We discussed the form of the cost functions investigated in a previous section. Here we discuss some practical aspects such as what independent variables are chosen and how the cost functions are evaluated since analytic functions are not available. To simplify some of the constraints and to provide a more intuitive approach to the problem solution, we use the Keplerian elements as independent variables. Therefore, the state vector for the formation has 18 independent variables. For now, the epoch is assumed to be Jan 01 2012 at zero hours. To avoid numerical problems, all angles are in radians and the semimajor axis is nondimensionalized on a_E , the semimajor axis of the Earth's orbit about the sun. Lower and upper bounds are placed on the Keplerian elements to restrict the search space to within known feasible bounds. For example, if the semimajor axis is not nearly equal to a_E , then the formation will have an undesirable secular drift with respect to the Earth. So we require that $.9a_E < a < 1.1a_E$. The complete upper and lower bounds applied are

$$.9a_E < a < 1.1a_E \quad (29)$$

$$0 < e < .0105 \quad (30)$$

$$0 < i < .021 \text{ rad} \quad (31)$$

$$0 < \omega < 2\pi \quad (32)$$

$$0 < \Omega < 2\pi \quad (33)$$

$$0 < M < 2\pi \quad (34)$$

$$(35)$$

All gradients and the Hessian matrix are approximated using finite differencing. A numerical approach is used to approximate the cost functions since analytic solutions are not available. To approximate the cost function, two years of ephemeris in one day time steps is used. The integrals in Eqs.(5 - 9) are approximated using a trapezoidal approach. Each cost function is normalized on a characteristic value to ensure the nominal solution is on the order of one.

In the next section we discuss the results of the optimization for each cost function. The solutions are discussed in light of LISA performance requirements and compared to the nominal solution.

RESULTS

In this section we discuss results for each of the five cost functions. A total of about 60 converged solutions have been found. The solutions presented here are representative of the solutions found to date. Because there are multiple constraints and multiple measures of performance, some solutions not presented here might have slightly better performance in one area while poorer performance in another, or require relaxing some of the constraints.

In Table 3 is statistics for the nominal orbit for a two year propagation. The data is provided to allow a comparison between the performance of the optimal solutions and the nominal orbit under perturbations.

Table 3: Statistics for the Perturbed Orbit (2 Year Propagation)

Variable Name	2 Leg Definition
L	$5.00e^6$ km
Δr	61,615 km
$\% \Delta r$	1.232 %
$\Delta \dot{r}$	12.072 m/s
\dot{r}	6.918 m/s

Results for cf_1

Recall that by optimizing according to the cf_1 performance measure, we are trying to reduce the difference in length, of two of the legs in the formation over the entire mission life. The orbital elements for the solution presented here are in Table 9 in Appendix 1. As expected the solutions all have nearly the same semimajor axis and eccentricity. The inclinations differ by about a tenth of a degree.

In figure 4 we see the evolution of some useful parameters of the formation over two years. In the upper right-hand plot, it is seen that the lengths of Leg12 and Leg31 are nearly equal for the mission life. As expected the rate of change Leg12 and Leg31 are also nearly equal. It is interesting to note that two of the angles are also nearly equal in time.

Table 4 shows the statistics associated with this particular optimal solution for cf_1 . The average leg length is about 5 million km. The maximum difference between the

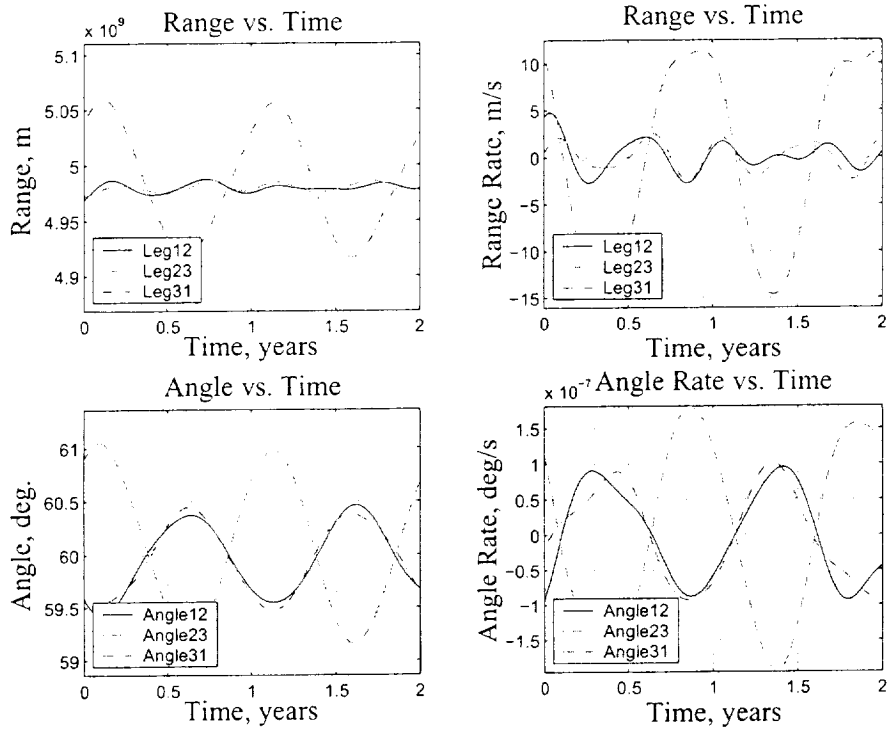


Figure 4: Results for cf_1

legs of interest is 5850 km and the maximum percent change, $\% \Delta r$, is .117%. This is an improvement of an order of magnitude over the nominal solution. For this solution all constraints are satisfied and inactive. The formation remains about 19° behind the Earth for the two year mission life.

Table 4: Statistics for Optimal Solution (cf_1)

Variable Name	2 Log Definition
L	$4.982e^6$ km
Δr	5850 km
$\% \Delta r$	0.117 %
$\Delta \dot{r}$	3.913 m/s
\dot{r}	14.625 m/s

Results for cf_2

By minimizing cf_2 we minimize the difference in the rate-of-change of two legs of the formation. The orbital elements for the solution presented here are in Table 10 found in Appendix 1. The statistics for the solution presented here are found in Table 5. The

average leg length is about 5 million km, and is Δr is 11,182 km. The $\% \Delta r$ is 0.226 % and $\Delta \dot{r}$ and \dot{r} are 2.185 m/s and 14.750 respectively. The evolution of the formation geometry is shown in Figure 5

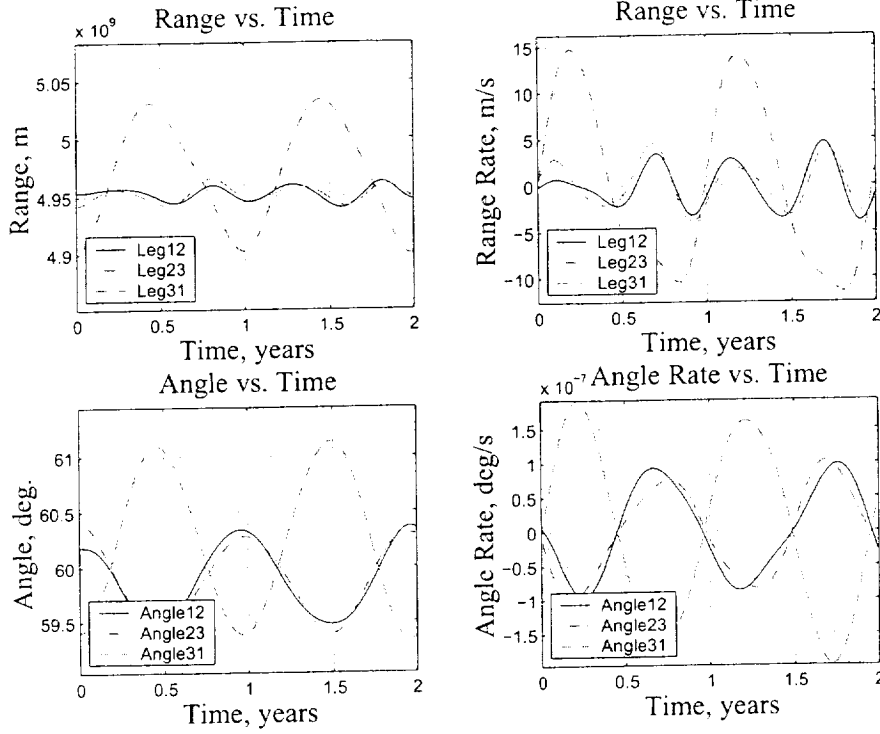


Figure 5: Results for cf_2

Table 5: Statistics for Optimal Solution (cf_2)

Variable Name	2 Leg Definition
L	$4.958e^6$ km
Δr	11,182 km
$\% \Delta r$	0.226 %
$\Delta \dot{r}$	2.185 m/s
\dot{r}	14.750 m/s

Results for cf_3

Table 11 contains the orbital elements for an optimal solution according to the cost function cf_3 . Recall that cf_3 is developed to minimize the average rate of change of the legs. A figure showing the evolution of the formation geometry is in Fig. 6. The case shown here is for a 5 year propagation so we must compare the statistics for this case

shown in Table 6 with the data for the nominal orbit with perturbations for a five year propagation shown in Table 2.

We see that the average leg length is about 4.93 million km. For the optimal solution, the maximum leg length is 61,442 km, while the maximum leg length is 132,300 for the nominal solution. The $\% \Delta r$ is 1.24% for the optimal case compared to 2.63% for the nominal case. Generally speaking, for the results of the cf_3 optimization, all of the statistics are reduced by a factor of two, as compared to the nominal solution.

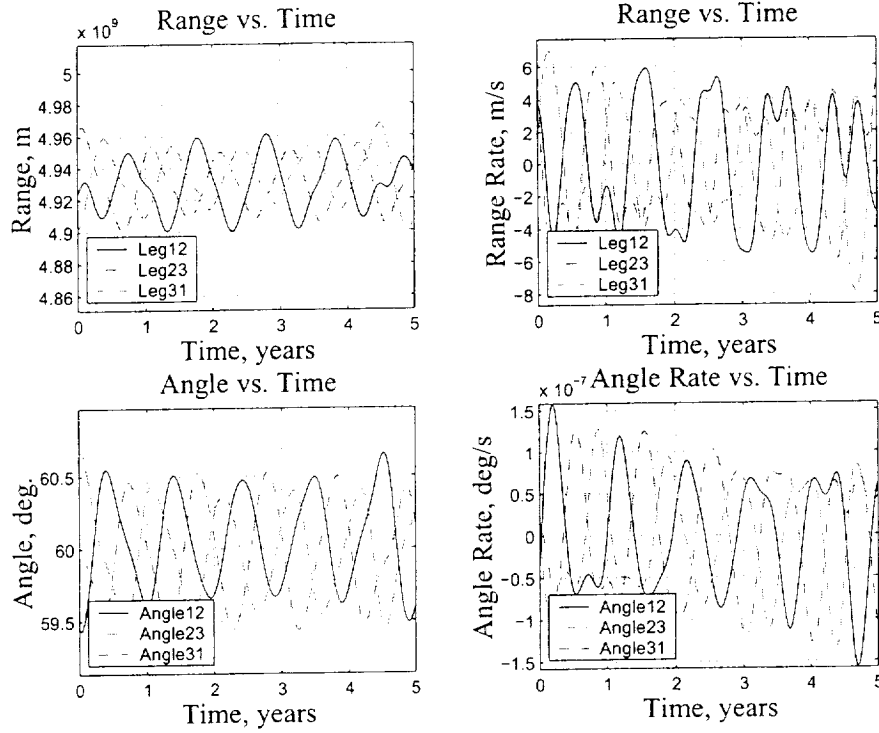


Figure 6: Results for cf_3

Table 6: Statistics for Optimal Solution (cf_3)

Variable Name	2 Leg Definition
L	$4.931e^6$ km
Δr	61442 km
$\% \Delta r$	1.246 %
$\Delta \dot{r}$	12.581 m/s
\dot{r}	7.869 m/s

Results for cf_4 and cf_5

Performance measures cf_4 and cf_5 are formulated to ensure all legs of the formation are equal in time. This is equivalent to a “breathing” equilateral triangle formation. To date no solutions that have acceptable performance and meet all of the design constraints have been found. Some solutions, which we present here, have been found by dramatically relaxing some of the design constraints.

In Figure 7 we see a solution obtained using the cf_5 cost function, where several of the constraints have been relaxed. Specifically, we have allowed the leg rates to vary up to 200 m/s and changed the lower bound on the leg lengths to 4 million km. Although the figure shows two years worth of data the optimization was only over six months. The leg lengths are nearly equal for the first six months. The statistics are shown in Table 7. The maximum difference in the leg lengths over the first 6 months is 7750 km and the $\% \Delta r$ is .160%. However, the maximum in leg rate of change is 120.25 m/s which is much higher than allowable. Furthermore, the system is highly unstable.

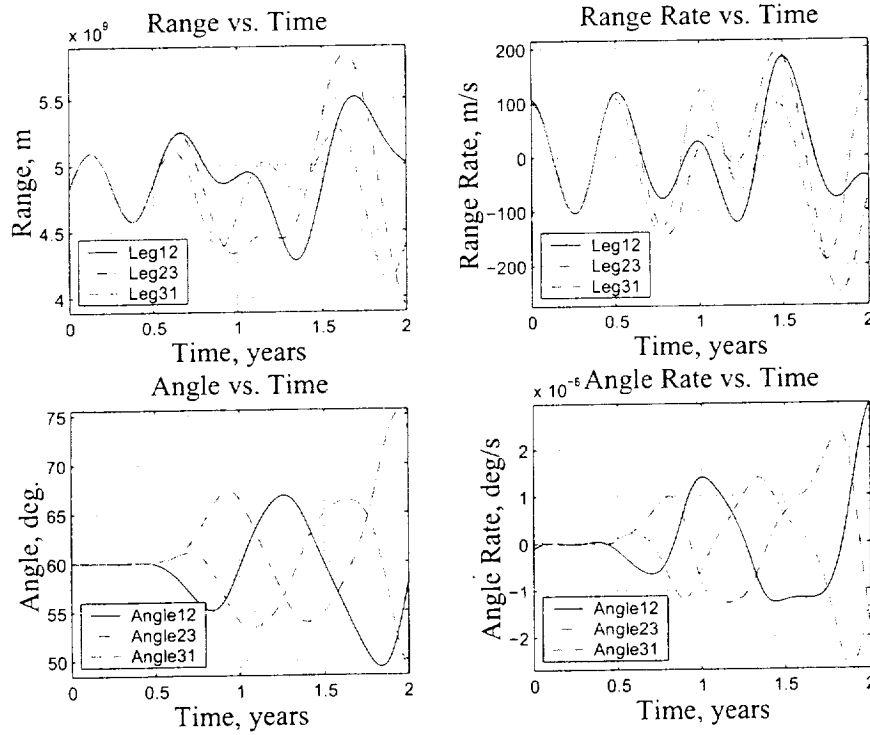


Figure 7: Results for cf_5

In Table 8 we see a plot of the evolution of a solution obtained using cf_4 . As in the previous example, we have allowed the leg rates to vary up to 200 m/s and changed the lower bound on the leg lengths to 4 million km. The average leg length for this particular solution is 4.2 million km. The maximum difference in leg lengths, Δr , is 34,055 km

Table 7: Statistics for Optimal Solution (cf_5), (6 Month Optimization)

Variable Name	3 Leg Definition
L	$4.843e^6$ km
Δr	7750 km
$\% \Delta r$.160 %
$\Delta \dot{r}$	12.16 m/s
\dot{r}	120.25 m/s

and $\% \Delta r$ is .817%. The maximum difference in leg rate of change is only 6.94 m/s, however, the maximum absolute leg rate is 42.02 m/s which is significantly higher than the acceptable value of 15 m/s.

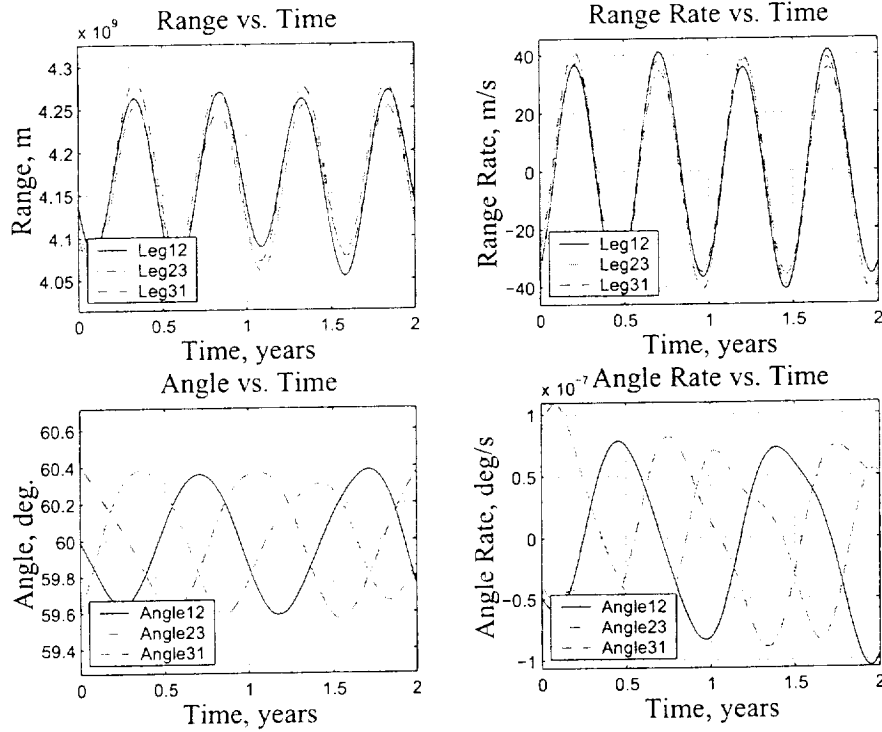


Figure 8: Results for cf_4

CONCLUSIONS AND FUTURE WORK

In this paper we presented optimal orbit configurations for the Laser Interferometer Space Antenna (LISA). The performance measures used are directly related to the science performance. Two categories of measures were considered. The first category investigated

Table 8: Statistics for Optimal Solution (cf_4)

Variable Name	3 Leg Definition
L	$4.169e^6$ km
Δr	34,055 km
$\% \Delta r$	0.817 %
$\Delta \dot{r}$	6.936 m/s
\dot{r}	42.016 m/s

functions only of two legs of the formation. The second category examined solutions that used all three legs of the formation in the performance metric. Several constraints were imposed on the solution. The leg lengths were required to be within two percent of the nominal, 5 million km length. The leg rates of change were constrained to be less than 15 m/s. Due to articulation limitations of the instrument, the interior angles of the formation were constrained to be $60 \text{ degrees} \pm 1.5 \text{ degrees}$. The formation was also constrained to remain within 20 degrees of the Earth as measured from a vertex at the Sun.

Feasible optimal solutions for category one performance measures were presented. About one order of magnitude of improvement in the measures were achieved over the nominal solution. Future work in this area involves a more thorough search of the design space for improved locally optimal solutions. Furthermore, relaxing the constraints may allow significant improvement in the performance.

It has proven difficult to find feasible optimal solutions for category two performance measures. Two optimal solutions were presented to illustrate possibilities if some of the design constraints can be relaxed. From experience gained so far, it is very difficult, if not impossible, to find feasible optimal solutions to category two measures within the current design constraints.

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APPENDIX 1

Table 9: cf_1 Results: Heliocentric Mean Equinox and Equator J2000 , 01 Jan 2012 00:00:00

OE	LISA1	LISA2	LISA3
a	149563305282.0	149563443458.0	149563663686.0
e	0.009346203767	0.009676975440	0.0098097142755
i	0.9438362726	1.000909246	0.9196112710
ω	86.37744956	92.20962395	91.05206384
Ω	217.3220002	333.4463294	92.13655101
ν	137.7874426	15.41162731	256.4720776

Table 10: cf_2 Results: Heliocentric Mean Equinox and Equator J2000 , 01 Jan 2012 00:00:00

OE	LISA1	LISA2	LISA3
a	152044712039.0	152044847970.0	152044235805.0
e	0.009392823346	0.009730991627	0.009519141055
i	0.8364363303	1.007377459	1.009488404
ω	90.78370457	84.18724496	94.71165172
Ω	173.4799425	299.1037480	48.20624884
ν	176.7385178	58.60738171	297.0484844

Table 11: cf_3 Results: Heliocentric Mean Equinox and Equator J2000 , 01 Jan 2012 00:00:00

OE	LISA1	LISA2	LISA3
a	149578965033.0	149580414155.0	149579800648.0
e	0.01034342962	0.009574892492	0.008682661929
i	1.023240970	0.9358205364	0.8785173418
ω	90.76228189	89.11274610	89.41129865
Ω	212.7394780	325.7412867	94.24688340
ν	136.5015274	24.79455200	254.5655484